Synopsis for Ph.D. and Registration

Faculty of Science (Mathematics)

Title of the Synopsis: Deterministic Dynamical Systems with Nonlinear Transition Rates: A Comprehensive Analysis

Work plan:

1- Introduction

Deterministic dynamical systems are essential to comprehending the qualitative and quantitative behavior of many biological, physical and natural phenomena. It is impossible for linear assumptions to accurately describe the complex interactions seen in many real-world systems. Rather, to explain the real dynamics, nonlinear transition rates such as density dependent growth, saturation incidence or nonlinear recruitment processes must be included. The objective of this work is to examine the long-term behavior of system variables under broad nonlinear transitions by establishing and assessing a deterministic nonlinear dynamical model. The aim of the paper is to provide a rigorous mathematical framework that is backed by numerical simulations and qualitative research.

2. Motivation of the Study

Nonlinearities arise naturally in epidemiology, ecology, population biology, chemical kinetics and many socio-dynamic systems. These nonlinear effects significantly influence equilibrium behavior, threshold parameters, system stability and possible bifurcations.
Understanding such systems is crucial for predicting outcomes, designing interventions and identifying critical thresholds that determine whether the system converges to stability, diverges or exhibits complex patterns. This research is motivated by the need for a unified analytic approach to study these nonlinear dynamical systems using deterministic differential equations.

3. Objectives

The major objectives include:

1. To construct a deterministic dynamical system with generic nonlinear transition functions.
2. To determine every systemic equilibrium point that is practicable.
3. To determine and examine the threshold conditions that control how the system behaves.
4. To implement standard mathematical methods to investigate the local and global stability of equilibria.

5. To glance at whether bifurcation events brought on by nonlinear transition terms occur.
6. To verify theoretical results using graphical representations and numerical simulations.

4. Chapter wise detail:

Chapter 1 - Introduction and Preliminaries

This chapter provides a broad introduction to deterministic dynamical systems and explains why nonlinear transition rates are essential in understanding real-world biological and epidemiological processes.

In Chapter 2-4 - This chapter constructs a deterministic nonlinear model for an infectious disease (or any biological process)

CHAPTER 5: Summary and Conclusions

This final chapter summarizes the entire research work.

5. Methodology

The proposed model will be formulated using ordinary differential equations (ODEs) with nonlinear transition terms such as:

- nonlinear incidence rates,
- nonlinear recruitment mechanisms,
- density-dependent removal or transition rates, and
- nonlinear interaction functions.

The methodological steps include:

- Mathematical formulation of the system.
- Non-negativity and boundedness analysis.
- Determination of equilibrium points.
- Calculation of basic threshold parameters.
- Local stability analysis via eigenvalues and Jacobian matrices.
- Global stability conditions using Lyapunov functions or geometric approaches.
- Bifurcation analysis (if applicable).
- Numerical simulations using MATLAB/Python to support analytical results.

6- Hypothesis

1. Nonlinear transition rates significantly modify the qualitative behavior of deterministic dynamical systems compared to linear models.

2. Equilibrium points and their stability are strongly affected when nonlinear transition mechanisms are incorporated.
3. The basic threshold conditions (such as the basic reproduction number) change when nonlinear terms are introduced.
4. Nonlinear deterministic models may exhibit bifurcation phenomena, influencing long-term system dynamics.
5. Models with nonlinear transitions provide more realistic and biologically meaningful predictions than classical linear models.
6. Numerical simulations will support and validate the analytical results obtained from stability and bifurcation analysis.

References:

1. Moghadas, M. Seyed and Abba B. Gumel. "Global stability of a two-stage epidemic model with generalized non-linear incidence." *Mathematics and computers in simulation,* 60 (2002) 107-118.
2. Saha, Pritam, Bapin Mondal and Uttam Ghosh. "Dynamical behaviors of an epidemic model with partial immunity having nonlinear incidence and saturated treatment in deterministic and stochastic environments." *Chaos, Solitons & Fractals,* 174 (2023): 113775.
3. He, Jian Jun, et al. "Stability analysis of a nonlinear malaria transmission epidemic model using an effective numerical scheme." *Scientific Report,s* 14 (2024) 17413.
4. Das, Riya, Dhiraj Kumar Das, and T. K. Kar. "Analysis of a chronological age-structured epidemic model with a pair of optimal treatment controls." *Physica Scripta,* 99 (2024) 125240.
5. Khan, Rehan, et al. "Deterministic and fractional-order modeling of measles dynamics with harmonic mean incidence rate and quarantine impact." *Scientific Reports,* 15 (2025) 30597.

(Name of Student)

Dr. Renu

(Name of Supervisor)

H.O.D
DEPTT. OF MATHEMATICS
UNIVERSITY
MADHEPURA